

Name KEY ID KEY

### Midterm Exam # 1 - 50 Points

The exam is closed book and closed notes. Please show your work step by step. Simple calculators may be used (no graphing calculators and no smart phones or iPods)

**You must show your work to receive full credit**

*I have neither given nor received unauthorized aid on this examination, nor have I concealed any similar misconduct by others.*

Signature KEY

#### Problem 1 (20 points)

- a.) Patrick Willis has played three games, getting 2, 0, and 1 sack in each. He also made 4, 3, and 5 tackles in each game, respectively. Please calculate the covariance between sacks and tackles. (10 Points)

$$\hat{\mu}_s = \frac{1}{3}(2+0+1) = 1 \quad +4$$

$$\hat{\mu}_T = \frac{1}{3}(4+3+5) = 4$$

$$\hat{\sigma}_{ST} = \frac{1}{3-1} \left( (2-1)(4-4) + (0-1)(3-4) + (1-1)(5-4) \right)$$

$$= \frac{1}{3-1} (0 + 1 + 0) \quad +4$$

$$\hat{\sigma}_{ST} = \frac{1}{2} \quad +2$$

b.) Suppose that you have a paired set of data,  $x$  and  $y$ , and that you define  $w = ax$  and  $z = -\frac{1}{a}y$ , where  $a > 0$ .

Please write the correlation of  $w$  and  $z$  as a function of the correlation of  $x$  and  $y$ . Though you may use results we discussed in class to simplify the answer, please detail how you obtain your answer. (10 points)

$$\hat{\rho}_{wz} = \frac{\hat{\sigma}_{wz}}{\hat{\sigma}_w \hat{\sigma}_z}$$

$$\left. \begin{aligned} \hat{\sigma}_w &= a \hat{\sigma}_x \\ \hat{\sigma}_z &= \frac{1}{a} \hat{\sigma}_y \end{aligned} \right) + 11$$

$$\left. \begin{aligned} \hat{\sigma}_{wz} &= a \cdot \left(-\frac{1}{a}\right) \hat{\sigma}_{xy} \\ &= -\hat{\sigma}_{xy} \end{aligned} \right) + 11$$

$$\Rightarrow \hat{\rho}_{wz} = - \frac{\hat{\sigma}_{xy}}{a \hat{\sigma}_x \frac{1}{a} \hat{\sigma}_y} = - \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\Rightarrow \hat{\rho}_{wz} = - \hat{\rho}_{xy} \quad + 2$$

**Problem 2 (30 Points)**

Suppose that hours of sleep per night is characterized by a normal distribution with mean 7 and standard deviation 1.

- a. What is the probability that a randomly selected person sleeps more than 6 hours per night? (10 points)

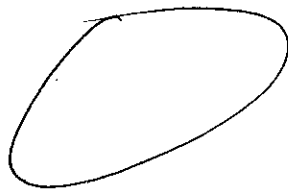
$$Pr(S > 6) = Pr\left(z > \frac{6-7}{1}\right) = Pr(z > -1)$$

$$\begin{aligned} +2 & \\ +6 & \left( \begin{aligned} &= 1 - Pr(z < -1) \\ &= 1 - (1 - Pr(z < 1)) \\ &= Pr(z < 1) \end{aligned} \right) \end{aligned}$$

$$\Rightarrow Pr(S > 6) = Pr(z < 1) = \underline{0.8413}$$

+2

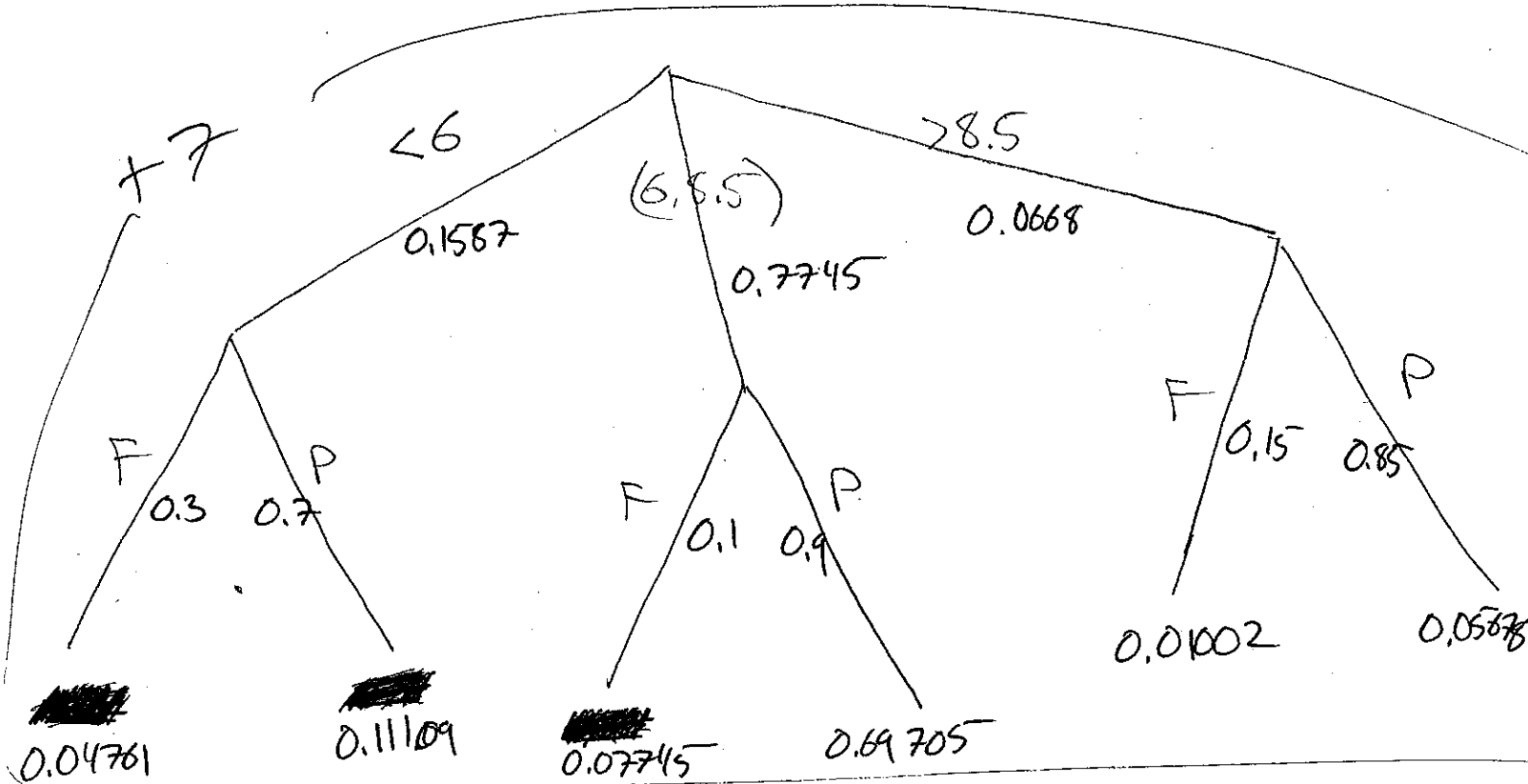
- b. Professor Spearot, hardly a sleep expert, claims that you must get exactly 8 hours of sleep per night to remain healthy. What is the probability of this happening? (5 points)



+5

all or nothing

- c. Doing well on a test is important, and sleep matters for doing well. Suppose that if you get less than 6 hours of sleep, the probability of failing is 0.3. If you get between 6 and 8.5 hours of sleep, the probability of failing is 0.1. If you get over 8.5 hours of sleep, the probability of failing is 0.15. Given that you have passed the exam, what is the probability that you got less than 6 hours of sleep? Please diagram this problem and show your work. (15 points)



$$Pr(S < 6) = 1 - Pr(S < 8) = 1 - Pr(Z < 1) = 1 - 0.8413 = 0.1587$$

$$Pr(S > 8.5) = 1 - Pr(S < 8.5) = 1 - Pr(Z < 1.5) = 1 - 0.9332 = 0.0668$$

$$Pr(6 < S < 8.5) = 1 - 0.1587 - 0.0668 = 0.7745$$

$$Pr(S < 6 | P) = \frac{Pr(S < 6 \cap P)}{Pr(S < 6 \cap P) + Pr(6 < S < 8.5 \cap P) + Pr(S > 8.5 \cap P)}$$

$$= \frac{0.11109}{0.11109 + 0.69705 + 0.05678}$$

$$Pr(S < 6 | P) = 0.1284$$



# Normal Distribution from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990